Finite size effects and the extrapolation of T_c to infinite nuclear matter

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The coexistence line and phase diagram obtained in [1] refer to well specified nuclei, which are finite systems. Our goal is to extrapolate these results to infinite nuclear matter.

Finite size effects are paramount in nuclei. For instance, the binding energy per nucleon decreases from the ~ 15.5 AMeV calculated for nuclear matter to about 8 AMeV for typical nuclei. This lowering of the binding energy is understood as due to the surface (and Coulomb) energy.

We can expect that such a drastic reduction affects the critical temperature as well. The Ising model can be used as a simple testing ground. As for nuclei, we have a volume energy. If a finite system is considered (with no periodic boundary conditions) a surface is generated with the attendant surface energy. This allows us to write a "liquid drop" formula for the Ising model:

$$E = a_V A + a_S A^{2/3}. \tag{1}$$

In most fluids for which a liquid drop expansion is applicable the volume coefficient a_V is approximately equal and opposite to the surface coefficient a_S: $a_V \approx -a_S$. This is true for nuclei and it is expected for the Ising model.

We now determine the critical temperature for lattices of various size Figure 1 shows a remarkable decrease of T_c with decreasing lattice size which we can understand in the following way. The infinite Ising model contains a single parameter, the interaction strength between nearest neighbors, that determines the energy scale and thus the binding energy per site. We now naively guess that, for a finite system, there is an "effective" interaction strength which results in an "effective" binding energy per site which can be accounted for by the surface energy. We can write

$$\frac{T_c^{A_0}}{T_c^{\infty}} = \frac{a_V A_0 + a_S A_0^{2/3}}{a_V A_0} = 1 - \frac{1}{A_0^{1/3}} = 1 - \frac{1}{L} \tag{2}$$

where A_0 is the number of sites in the lattice and L is the length of on side of the lattice. This naive version of the finite size scaling of the critical temperature has been discussed long ago [2] and more sophisticated versions have been theorized [2] and observed such as [3,4]

$$\frac{T_c^{A_0}}{T_c^{\infty}} \propto 1 - L^{-1/\nu} \tag{3}$$

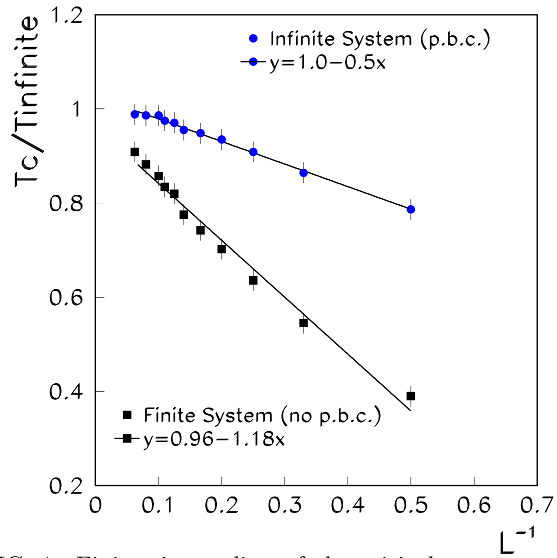

FIG. 1. Finite size scaling of the critical temperature of the three-dimensional Ising model. Left: the naive estimate of finite size scaling. Right: the sophisticated estimate of finite size scaling. The data points and fits on the top of both figures show the results for lattices with periodic boundary conditions (p.b.c.) which more closely represent an infinite system. The data points and fits on the bottom of both figures show the results for lattices with open boundary condition (no p.b.c.) and more closely represent the case of finite systems like nuclei.

where ν is the critical exponent describing the divergence of the correlation length near the critical point. For the three-dimensional Ising lattice $\nu \approx 0.63$. However, for the purposes at hand, the naive finite size scaling is sufficient and the sophisticated version produces essentially equivalent results to those shown in Figure 1.

The result of this exercise is to show that the critical temperature of infinite nuclear matter should be approximately equal to that of the finite nucleus times the ratio of the binding energy of infinite nuclear matter to the binding energy of the finite nucleus.

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